Calculating the Number of Atoms in the Sun

G. H. Brown

Introduction

The feat described in the title is said to have been attempted (if not completed) by Freeman Dyson at the age of 4 or 5. While the task is not difficult, it takes some contemplation and knowledge of basic chemistry/unit analysis. Dyson's approach is unknown, but here the calculation will proceed as follows: find a general formula to calculate the number of atoms in any object using knowledge of its total mass and elementary composition by mass, then apply this method to the Sun.

Developing the Equation

The total number of atoms in any object may be written as the sum of the number of atoms of each constituent elements. Mathematically this takes the form

$$N_{tot} = \sum_{e=1}^{T} N_e \tag{1}$$

where N_{tot} is the total number of atoms, N_e is the number of atoms of element e, and the total number of elements in the object is T. Because it is useful when dealing with very large quantities of things, such as atoms, the concept of the mole will be introduced. One mole of any "thing" is defined as $6.022 \cdot 10^{23}$ of that "thing"; for example, one mole of hydrogen atoms is $6.022 \cdot 10^{23}$ hydrogen atoms. This relationship is contained nicely in Avagadro's Number, $\mathscr{N}_A = 6.022 \cdot 10^{23}$ [mol^{-1}].

Determining the number of atoms of element (e) in an object is not easy, however. Therefore, the variable N_e in Equation (1) should be manipulated into a more useful form. To do this, is it most simple to start from mass, a more easily measured property. The molar mass M_e of element e simply relates the number of moles and the mass. This relationship is

$$m_e = n_e M_e \tag{2}$$

where m_e is the mass of some collection of e atoms, n_e is the number of moles of element e in the collection, and M_e is the molar mass of element e. Molar mass reports the mass in one mole of a substance and so has units of mass per mole. Here we will take molar mass to to have units of [kg$\cdot$mol^{-1}]. Equation (2) may be rewritten as

$$n_e = \frac{m_e}{M_e} \quad \text{or} \quad n_e = m_e M_e^{-1}. \tag{3}$$

Continuing with the second form of Equation (3), each side can be multiplied by $\mathscr{N}_A$ to convert from [mol] to [atoms] (n_e to N_e).

$$N_e = m_e M_e^{-1} \mathscr{N}_A \tag{4}$$

Looking at Equation (1) and Equation (4) note that we now must find estimates for the mass of each element in the object, rather than the number of atoms of each element. However, the total mass of each element in an object is not easily found; furthermore, it varies for similar objects of different sizes. For example, two people of drastically different heights will certainly have different values for m_{oxygen}. Since the notion of percent composition is more readily available ("the human body is 65% oxygen (O) by mass", etc.), that concept will be used to replace m_e and further generalize the formula we are constructing. Percent composition by mass is defined as

$$f_e = \frac{m_e}{m_{tot}} \tag{5}$$

where f_e is the fraction of the body (by mass) which is comprised of element e and m_{tot} is the total mass of the object. Therefore, m_e can be written as

$$m_e = f_e m_{tot}. \tag{6}$$

Finally, the formula for the number of atoms in an object can be written in a usable form. Substituting Equation (5) into Equation (4), and that combination into Equation (1)

$$N_{tot} = \sum_{e=1}^{T} f_e m_{tot} M_e^{-1} \mathscr{N}_A. \tag{7}$$

Since m_{tot} and $\mathscr{N}_A$ are clearly independent of e, Equation (7) can be rewritten as

$$N_{tot} = m_{tot} \mathscr{N}_A \sum_{e=1}^{T} f_e M_e^{-1}. \tag{8}$$

Calculation

With Equation (8) developed, the total number of atoms in the Sun (N_{Sun}) can now be calculated. Values of f_e and M_e for the five most abundant elements can be found in Table 1, below. The mass of the Sun used for this calculation was $1.989{\cdot}10^{30}$ [kg]. Using these values and Equation (8), **the total number of atoms in the Sun is calculated to be $9.26{\cdot}10^{56}$ [atoms].**

Table 1: Values used for calculating N_{Sun} (only including the five most abundant elements).

Element	f_e	M_e [kg mol^{-1}]
H	0.71	$1.008{\cdot}10^{-3}$
He	0.271	$4.003{\cdot}10^{-3}$
O	0.0097	$15.999{\cdot}10^{-3}$
C	0.004	$12.011{\cdot}10^{-3}$
Fe	0.0014	$55.845{\cdot}10^{-3}$