The Theoretical Minimum Time for Competition Speed Climbing

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Introduction

Competition speed climbing is an athletic event in which climbers attempt to scale a 15 meter (nearly) vertical wall in the minimum possible time. The hold shapes and placements are identical between walls across the world. [1] The current world record for speed climbing is 5.48 seconds, but by using a simple energy based model, it is possible to compute the theoretical minimum time for the completion of this event by humans.

Model

The current model for this event considers the climber as a point mass and ignores the dynamics of the climb, tracking instead the total energy of the climber system. The main equation will be the conservation of energy

$$E(t_0) = E(t_1) \tag{1}$$

which states that the energy at t_0 plus is equal to the energy at a later time t_1. In this case we will consider only gravitational potential energy and chemical potential energy. The reason that kinetic energy may be ignored is that the climber begins at rest, and (in most videos of climbers finishing the event) are nearly at rest since the apex of their final leap approximately coincides with them hitting the timing plate. Making these simplifications the above equation becomes

$$U_g(t_0) + C(t_0) = U_g(t_1) + C(t_1) \tag{2}$$

where the U_g terms are the gravitational potential energy and the C terms are the chemical potential energy. By choosing the reference point of potential energy to be at the location of center of mass of the climber at t_0, $U_g(t_0) = 0$. Further, by considering that the climber uses up all of some amount (we do not yet know how much) of their chemical potential energy during the climb, we can consider $C(t_1) = 0$. [2] Making these further simplifications, one has

$$C = U_g(t_1) \rightarrow \int_{t_0}^{t_1} P(t)dt = mgh(t_1) \tag{3}$$

where we have substituted in convenient forms for these energies. The climber's potential energy is transformed into their gravitational potential energy via a time dependent power (peaking when the climber exerts themself, and zero when remaining still), and we have used the standard formula for gravitational potential energy near the surface of the Earth which scales linearly with height above the reference point. The final simplification is to convert from the integral to the average power times the time difference $t_1 - t_0$. This is a reasonable approximation since the climbers appear to be expending themselves at a constant rate

[1] More information can be found here: `https://en.wikipedia.org/wiki/Speed_climbing`.

[2] A more rigorous (but equivalent) way to arrive at this potential energy term would be to consider the total amount of chemical energy available to the climber at t_0 and call that $C_{total}(t_0)$. Now partition this total amount of chemical energy into two parts, the part used in the climb and the part leftover for the climber to do stuff after the climb as $C_{total}(t_0) = C_{climb} + C_{leftover}$. We don't know how much is in each partition, but we do know that after climbing the chemical potential energy in the climber will be $C_{leftover}$. Inserting $C_{total}(t_0)$ on the left and $C_{leftover}$ on the right, the leftover terms will cancel, leaving only C_climb, which is what I have called C in the main body.

througout the climb, they do not (for example) start very slowly and gradually ramp up speed throughout the climb.

While not a simplification, the last point to consider is the value $h(t_1)$. Naively, this value is the height of the timing plate from the ground (often called the height of the wall/event: 10 meters, 15 meters (the standard), etc.). However, one should in fact subtract the standing reach of the climber from the wall height to find $h(t_1)$. [3] Making these final adjustments, we have

$$P_{avg}t = mg(h_{wall} - h_{SR}) \tag{4}$$

which can be solved easily for the climb time t.

Calculations and Estimates

Given Equation (4), we need only substitute in best case estimates for human mass, standing reach, and power output to find the minimum theoretical time for the 15 meter climb. It's clearly unrealistic to optimize each of these parameters independently (for example humans of lower mass are likely to have a lower standing reach and be able to sustain lower power outputs), we will use the standing reach and mass of the current world record holder, while letting P_{avg} be the largest known power output of a human. [4] [5]

The current record holder is Reza Alipour, who has a mass of 76 kilograms and stands at 1.7 meters tall (giving him an approximate standing reach of 2.2 meters). [2] The largest known human power output at the time of publication is a peak of 2600 Watts by Usain Bolt. [6] [3] Assuming Reza was able to continuously output Bolt's peak power throughout his climb the theoretical minimum time for the 15 meter climb is 3.67 seconds. [7]

However, it is important to think for a moment about what this number implies. The circumstances of such a rapid climb imply that the current world record holder in a popular athletic event could increase his effective average power output by about 50%, sustaining the largest peak human power output of a human. This is extraordinarily unlikely, so a more realistic (but not necessarily strict) lower bound on the climb time for this event might consider Reza (with all other things being equal) being able to output 2000 Watts on average. This more realistic figure would given a minimum climbing time of 4.77 seconds. [8]

[3] Consider why the naive height is wrong with an absurd example. How far would a climber with a standing reach just long enough to touch the timing plate while standing on the ground have to climb? Zero meters. Climbers' centers of mass do not travel the full height of the wall; their centers of mass start a little bit off of the ground and end a little bit below the timing plate (which defines the wall height). It turns out the these two "bits" are equal to the standing height of the climber when added together.

[4] If one knew the interdependencies of these parameters of these three variables, one could actually predict what sort of body types, masses, heights, etc. were likely to succeed in this event.

[5] Further, our approach of using the current world record holder's biometrics is reasonable, as it is likely their body composition and metrics are highly suitable to this event.

[6] This value is within reason (but unlikely sustainable), considering that cyclists have been known to push over 2000 W in short burst, and when back calculating the average power of Reza's record run one finds about 1740 Watts. Some examples of larger power outputs (though not relevant in the more dynamic event of speed climbing) could include powerlifting squats or similar movements.

[7] I am claiming this as a strict lower bound, since (as I discuss more in the main body) the circumstances leading up to such a climb are extraordinarily unlikely. As such, I explicitly claim that no (non-doping, non-genetically modified) human climbs will ever occur in less time.

[8] Certainly factors like route/hold choice, etc. are other possible optimizations which could lower the time, but the simple model used here has no concept of or dependence on such things, and such things should not be necessary to compute a minimum time.

References

[1] International Federation of Sport Climbing, 2018, “IFSC Climbing World Cup Nanjing 2017 - New Speed World Records”, from url https://www.youtube.com/watch?v=xCWUEQEy7x8.

[2] “Reza Alipour-Wikipedia”, from url https://en.wikipedia.org/wiki/Reza_Alipour

[3] Gomez et al., 2013, “On the performance of Usain Bolt in the 100 metre sprint”, *European Journal of Physics*, **34**(5), p. 7.